The general equation of motion via the special theory of relativity and quantum mechanics

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ABSTRACT. Herein we present a whole new approach to the derivation of the *Newton's Equation of Motion.* This, with the implementation of a metric imposed by *quantum mechanics,* leads to the findings brought up within the frame of the general theory of relativity *(such as the precession of the perihelion of the planets, and the deflection of light nearby a star).* To the contrary of what had been generally achieved so far, our basis merely consists in supposing that the gravitational field, through the *binding process,* alters the *"rest mass"* of an object conveyed in it. In fact, the special theory of relativity already imposes such a change. Next to this fundamental theory, we use the classical Newtonian gravitational attraction, reigning between two *static masses.* We have previously shown however that the $1/r^2$ dependency of the gravitational force is also imposed by the special theory of relativity.

Our metric is *(just like the one used by the general theory of relativity)* altered by the gravitational field *(in fact, by any field the "measurement unit" in hand interacts with);* yet in the present approach, this occurs via *quantum mechanics.* More specifically, the *rest mass* of an object in a gravitational field is decreased as much as its *binding energy* in the field. A *mass deficiency* conversely, via *quantum mechanics* yields the *stretching* of the size of the object in hand, as well as the *weakening* of its internal energy. Henceforth one does not need the *"principle of equivalence"* assumed by the general theory of relativity, in order to predict the occurrences dealt with this theory.

Thus we start with the following interesting postulate, in fact nothing else, but the *law conservation of energy,* though in the broader relativistic sense of the concept of *"energy".*

Postulate : The *rest mass* of an object bound to a celestial body amounts less than *its rest mass measured in empty space,* and this as much as its *binding energy* vis-à-vis the *gravitational field* of concern.
This yields *(with the familiar notation),* the interesting *equation of motion driven by the celestial body of concern,* i.e.

$$\frac{e^{-\alpha_0(r_0)}}{\sqrt{1-\frac{v_0^2}{c_0^2}}}=\text{Constant};\ \alpha_0(r_0)=\frac{G\mathrm{M}_0}{r_0c_0^2};$$

here M_0 is the mass of the celestial body creating the gravitational field of concern; G is the universal gravitational constant; r_0 points to the location picked up on the *trajectory* of the motion; v_0 is the tangential velocity of the object at r_0 ; c_0 is the speed of light in empty space.
The differentiation of this relationship leads to

$$-\frac{G\mathrm{M}_0}{r_0^2}\left(1-\frac{v_0^2}{c_0^2}\right)=v_0\frac{dv_0}{dr_0}.$$

This differential equation is the *classical Newton's Equation of Motion,* were v_0, negligible as compared to c_0 *(the speed of light in empty space).*
The streching of lengths in a gravitational field, is equivalent to the slowing down of light, throughout, as referred to a distant observer. Based on this, the above differential equation can be transformed in regards to the distant observer. The mathematical manipulation in question, together with the related solution, will be undertaken in our next article.

1 Introduction

Herein we present a whole new approach to the derivation of the Newton's Equation of Motion, as well as the findings brought up within the frame of the general theory of relativity (*such as the "precession of the perihelion of the planets", and the "deflection of light nearby a star"*).

To the contrary of what had been generally achieved so far, the basis adopted herein merely consists in supposing that the gravitational field, through the binding process, alters the "rest mass" of an object conveyed in it. In fact, the special theory of relativity astonishingly, far and wide overlooked, imposes such a change. Next to this fundamental theory, we use the classical Newtonian gravitational attraction reigning between two static masses. We have previously shown however that the $1/r^2$ dependency of the gravitational force is also imposed by the special theory of relativity [1]

Furthermore the metric coming into play in this work is (*just like the one used by the general theory of relativity*) altered by the gravitational field (*in fact, by any field the "measurement unit" in hand interacts with*); yet in the present approach, this occurs via quantum mechanics. In effect, the solution of even a non-relativistic quantum mechanical description, given that "potential energies existing in nature" are considered, bears a casing, in perfect harmony with the special theory of relativity. This is to say, regarding the internal dynamics of a wave-like object, "space" (*i.e. the size of the object), "time" (period of the internal dynamics of concern*), and "mass" (*the mass, to be associated with the wave-like object, working as the "pendulum mass" of its internal dynamics*), are structured in such a way that their interrelation remains Lorentz invariant (*i.e. invariant, were the object brought into a uniform translational motion*).

Thus, as we shall see, based on the special theory of relativity, the rest mass of an object in a gravitational field should decrease as much as its binding energy in the field; a mass deficiency conversely, via quantum mechanics, yields a stretching of its size, as well as the weakening of its internal energy (this is how the metric coming into play is altered by the field).

Therefore the basis of the approach undertaken herein, shrinks down to only the special theory of relativity.

Henceforth one does not need the "principle of equivalence" assumed by the general theory of relativity, in order to predict the occurrences dealt with this theory [2] We predict them through the general equation of motion established herein (thus, essentially based on the special theory of relativity, only).

Mass of the Bound Electron

A change, through the binding process, in the rest mass of an object interacting with a gravitational field, seems somewhat, clear, as the special theory of relativity predicts such an occurrence. For example, the proton and the electron, when bound to each other in the hydrogen atom, weigh less than the sum of the proton and the electron, carried away from each other; the mass deficiency in question is (by taking the speed of light, unity), exactly equal to the binding energy of the proton and the electron in the hydrogen atom, i.e. 13.6 ev, based on the fundamental relationship [3]

(Energy released, or acquired) =
(Magnitude of the algebraic increase in the mass)
x (Speed of light in empty space)^2. (a)

So, on the contrary to the widespread opinion, the *electron* or the *proton* cannot be the *same,* when bound to each other; they are different. Their internal dynamics altogether, *weaken* as much as 13.6 ev, when they are bound to each other to shape up the hydrogen atom.

Many scientists though, still firmly think that there is the *"proper mass" (rest mass)* and the *"relativistic mass" (defined within the frame of the special theory of relativity),* and that the *proper mass* is, whatsoever, an invariant which is a *characteristic of matter,* and that is all.

Generally speaking, this is unacceptable. The proper mass of a given particle on the whole at rest may, depending on the circumstances, embodies a more or less energetic internal motion; this will, one way or the other, affect the proper mass.

Suppose indeed that Captain Electron *(we mean, the electron itself)* is cruising in a *full electric isolation,* with a uniform translational velocity. So does Captain Proton *(i.e. the proton itself).* They approach to each other. Then *(based on the special theory of relativity)* we would be certain that, Captain Electron in its own frame of reference, all the way through, preserves its identity, defined at infinity. *(So will also do Captain Proton.)* If now, we remove the *previous electric isolation,* Captain Electron and Captain Proton, because of the electric attraction force, they mutually create, shall start getting accelerated toward each other. The *"extra kinetic energy"* they would acquire, as well as the energy they would radiate through this process, ought to be supplied by the system made of the two. *(For easy wording, we will neglect the energy emitted by radiation, without though any loss of generality Note that the radiation energy is anyway negligible.)* The total energy of Captain Electron and Captain Proton [i.e. (the sum of their relativistic masses) x (the speed of light)^2], through the motion, must remain *constant,* and *equal* to the equivalent of the sum of their initial relativistic masses. *(Otherwise, the energy conservation law would be broken.)* Let us suppose for simplicity that in the latter case *where we have no electric isolation,* they start, far away from each other, *at rest;* then their initial relativistic masses are, essentially equal to respectively their rest masses. If now the *accelerating Captain Electron,* say in Captain Proton's frame of reference, hurts an *obstacle* and looses all the kinetic energy, it would have acquired through the attraction process; it must *concurrently* dump a portion of its rest mass, and this, as much as the amount of the kinetic energy it would have piled up, on the way.(1)

Thus, we cannot say that *the proton and the electron are the same, after we have retrieved from the system made of the two, a given amount of energy,* no matter how much. The greater is the energy extracted, the harder will be the harm caused in their internal dynamics, consequently in their proper masses defined at infinity.

This is exactly what happens when, say the hydrogen atom is formed, except that the electron, as referenced to the proton is not anymore at rest, but possesses a given amount of kinetic energy; still an energy of 13.6 ev is needed, to carry the electron away from the proton, back to infinity.

It is thus clear that in this case, as referenced to the proton, or since the proton is much too big as compared to the electron, practically the same, as referenced to the laboratory system, the hydrogen electron's proper (rest) mass, must be increased as much.

Likewise, the daily production of thermal energy, is due to the transformation of a minimal part of the fuel mass entering in reaction, into energy. Thus the reaction products weigh less than the reactants, and this, as much as the energy produced throughout.

The fuel, i.e. coal, petroleum, uranium, plutonium, anything, in a power plant of, say 3000 $MW_{thermal}$, continuously working for a period of one year, thus producing an energy amounting to 3000 $MW_{thermal}$ x year, at the end of this period, weighs less, and this as much as the equivalent of the energy output in question, i.e. [based on the equivalence between mass and energy], about 1 kg. This is of course insignificant as compared to millions of tons of coal or petroleum that would be fired into such a plant, but is well detectable as compared to about a ton of plutonium-239, or uranium-235 needed to be depleted in a nuclear power plant of the same size, through a period of one year.

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(1) It was an incomparable privilege to have discussed with Professor R. Feynman, the very first seed of the idea presented herein, and to have been encouraged with his support, through a Fulbright visiting stay at California Institute of Technology, back in 1984. It is also a privilege to have been recently backed up along the same line by Professor Rozanov, Director of Laser Plasma Theory Department of Lebedev Institute, Moscow.

In a similar way, a compressed spring should be heavier than the "same spring" when stretched out; or the gas in a room at a high temperature should weigh more than the "same gas" at a lower temperature, etc.

To us all these, already happen to be well established facts. Thus, any proper mass weighs less, after releasing energy, or conversely it shall weigh more, after piling up an extra amount of internal energy.

Nuclear Binding Energy

Recall that in the case of a *nuclear fission*, we just referred to, or the same in *nuclear fusion*, it is the *increase in the binding energy* of the nucleons in the nuclei of the fission products, *in comparison with the binding energy of the nucleons in the original plutonium-239, or uranium-235 nuclei, in question*, which is responsible *of the nuclear energy released*; thence the *nuclear binding energy has well a mass deficit counterpart*.

Along this well known fact, in almost all related text books the following relationship is provided for the mass of the atomic nucleus:[4-8]

$$M_{Nucleus} = ZM_{Proton} + (A\text{-}Z)M_{Neutron} - E_B \text{ (the mass of the nucleus);} \quad \text{(b)}$$

Here $M_{Nucleus}$ is the *total mass of the nucleus* of concern; Z is the *number of protons* of this nucleus; A is the *total number of nucleons* residing in it; M_{Proton} is the mass of the *unbound proton*; $M_{Neutron}$ is the mass of the *unbound neutron*, and E_B is the *total binding energy* of the nucleus, i.e. the energy one has to furnish in order to dissociate it, into Z protons and (A-Z) neutrons.

Just likewise, the "*atomic binding energy*" must have a *mass deficit counterpart*, making that an *electron* bound to the atomic nucleus, must weigh less then the *free electron*.

Accordingly the *mass* of the atom should be described by a relationship similar to Eq. (b):

$$M_{Atom}=M_{Nucleus}+Zm_{e0}-E_B \text{ (the mass of the atom);} \quad \text{(c)}$$

Here M_{Atom} is the *total mass of the atom* in hand; $M_{Nucleus}$ is the *overall mass* of its nucleus, alone; Z is the *number of its electrons,* or the same the number of the protons residing in the nucleus, m_{e0} is the mass of the free electron, and E_B is the *total electronic binding energy* of the atom, i.e. the energy one has to furnish in order to dissociate this atom, into Z electrons and the nucleus.

It is astonishing that Eq.(c) can be found nowhere we could reach.

Note further that recently a *fluid model* of the bound electron is proposed, incorporating a change of the mass of the electron (*though in a totally different manner than the one proposed herein, thus*) through an exchange of mass between the electron and the nucleus [9]. (*A list of related interesting work, chiefly based on the more than a century old, Weber's Force Law, can be found amongst the references cited in this article*).

One way or the other, to us it seems unconceivable, not to associate a *mass deficit* with the *bound electron*.

Postulate Regarding an Object Bound to a Celestial Body, or the Law of Conservation of Energy

Based on the foregoing discussion, we anticipate that when an object is bound to a celestial body, its rest mass (measured in empty space) is decreased as much as the binding energy, it would have developed in the gravitational field of the celestial body of concern [10].

Einstein in his general theory of relativity, considers the conservation of the "rest masses", instead of the conservation of the "total energy" [2].

Yılmaz somewhat fulfilled this gap. He derived the "exact solution" of the "accelerated elevator", and to his great surprise, found out that Einstein's fields equations were not satisfied; this was the beginning of Yılmaz's efforts towards a more consistent theory, though along the same direction as that drawn by Einstein [11-13].

At any rate, Einstein's general theory of relativity leads to the fact that, his original relativistic *"mass-energy relationship"*, i.e. [Eq.(a)], does not hold between *values of energy and mass at different gravitational coordinates,* [14]. We do not have such an annoyance, since we derived our results essentially based on Einstein's *"mass-energy relationship",* i.e.

$$\Delta E \text{ (Energy released, or piled up)} = \Delta m \text{ (Magnitude of the algebraic increase in the mass) x } c_0^2 \qquad (1)$$

obtained within the frame of just the special theory of relativity (*and not the principle of equivalence assumed by the general theory of relativity*).

Thence one can propose the following postulate, in fact nothing else, but the *energy conservation law,* where though, now "*energy*" and "*mass*" are essentially no different from each other.

Postulate: The *rest mass* of an object bound to a celestial body, amounts less than its *rest mass measured in empty space,* and this as much as its *binding energy* vis-à-vis the *gravitational field* of concern.

It is important to note that, on the contrary to what the general theory of relativity eventually formulates, as we shall see, here, it is question of a *decrease of mass* in a gravitational field, and this is interestingly, just as much as the *mass increase (due to the equivalent acceleration)* formulated by the former theory.

So far there had been no measurement of mass in a gravitational field; thus the measurement of a mass embedded in a gravitational field, can furnish a verification of our guess.

Anyhow, based on our approach, the *classical red shift due to gravitation,* is nothing else, but an overall mass decrease, of the emitter embedded in the gravitational field, whereas this is due to the clock retardation process within the frame of the general theory of relativity. (Note that in our approach, as we will soon see, the mass decrease and the clock retardation processes due to gravitational binding, are simultaneous phenomena).

Below, we first sketch how the gravitational binding energy reduces the rest mass of an object bound to the celestial body in consideration (Section 2). Then, we recall the quantum mechanical theorems we have established previously (Section 3). An elaboration on the gravitational binding energy follows (Section 4). The change of the rest mass of an object in a gravitational field, together with the Lorentz mass dilation, due to the local motion, yields our general equation of motion (Sections 5 and 6). A conclusion follows (Section 7).

Next, taking into account how unit lengths, *quantum mechanically* stretch in a gravitational field, one is able to obtain the *precession of the perihelion of Mercury (or anything as such),* as well as the *deflection of light* grazing a celestial body; this shall constitute the content of our next article.

2 The gravitation binding energy

At this stage, we have to evaluate the *gravitational binding energy.* For this purpose we have to use the expression for *the gravitational force.*

Herein we consider *only* the *gravitational for*ce between two *static masses.*

Since we aim ultimately at deriving a result obtained within the context of the general theory of relativity, however *without having to rely on it,* we better should not even plainly borrow, the expression for the gravitational force *(between two static masses),* with its *classical empirical form,* from Newton, [15] since this *(were this the case of a weak gravitational field),* is formally, well manufactured by the general theory of relativity.

Therefore *(and luckily)* we derive the $1/r^2$ dependency of the gravitational force between two static masses, here again, from the special theory of relativity.[1]

Hence, one can calculate the binding energy E_B, of a given object in the gravitational field of the celestial body of concern, in the usual way. As a first approximation, let us consider that the binding energy is *small* as compared to the mass of the object. Thus

$$E_B = \int_{R_0}^{\infty} G\frac{M_0 m_0}{r^2}\,dr \cong G\frac{M_0 m_0}{R_0}; \quad (2)$$

here M_0 is the mass of the *host body* binding the object of mass m_0, as measured in empty space, R_0 the distance of the mass m_0 to the center of the host body, and G the *universal gravitational constant,* all of these quantities being defined (*as it will become clear soon*) in the *local frame of reference*.

In reality m_0 (based on the discussion presented above, in Section 1), changes continuously throughout. One can, as we shall soon see, easily elaborate on this.

When the object of mass m_0 is bound to the gravitational field, m_0 decreases to become m.

Thus

$$m_0 \rightarrow m, \quad m = \chi\, m_0; \quad (3)$$

χ is determined out of Eq.(1), more specifically

$$(m_0 - m)\,c_0^2 = E_B \quad (4)$$

Given E_B, χ *(smaller than unity)* becomes:

$$\chi = 1 - \frac{E_B}{m_0 c_0^2} = 1 - \frac{GM_0}{R_0 c_0^2}\ . \quad (5)$$

3 Theorems previously established

The approach presented herein becomes very interesting, if one recalls the following theorem established elsewhere [16-22].

Theorem 1: In a *"real wave-like description" (thus, not embodying artificial potential energies),* composed of I electrons and J nuclei, if the *(identical)* electron masses m_{i0}, i = 1,..., I and different nuclei masses m_{j0}, j = 1,…, J, involved by the object, are overall multiplied by the *arbitrary number* χ, then *concurrently, a)* the *total energy* E_0 associated with the given clock's internal motion of the object, is increased as much, or the same, the *period* T_0, of the motion associated with this energy, is decreased as much, *and b)* the *characteristic length* or the *size* R_0 to be associated with the given clock's motion of the object, contracts as much; in mathematical words this is

$$\{ [(m_{i0}, i = 1,..., I) \rightarrow (\chi m_{i0}, i = 1,..., I)], [(m_{j0}, j = 1,…, J) \rightarrow (\chi m_{j0}, j = 1,…, J)] \} \Rightarrow \{ [E_0 \rightarrow \chi E_0], [T_0 \rightarrow \frac{T_0}{\chi}], [R_0 \rightarrow \frac{R_0}{\chi}] \} . \quad (6)$$

Then, following the above derivation, we come at once, to the next theorem.

Theorem 2: A wave-like clock in a gravitational field, *retards* via *quantum mechanics,* due to the *mass deficiency* it develops in there, and this, as much as the binding *energy* it displays in the gravitational field; at the same time and for the same reason, the *space size* in which it is installed, *stretches* as much.

This can further be grasped rather easily as follows. *The mass deficiency the wave-like object displays in the gravitational field weakens its internal dynamics as much*. Thence, we arrive at the two principal results, we just stated.

Note that, according to the approach presented herein, the *classical gravitational redshift* and a *related mass decrease,* occur to be *concomitant quantum mechanical effects.* Thus in fact, on the contrary to what the general theory of relativity ultimately considers, we expect a *mass decrease* in a gravitational field *(and not a mass increase).*

It is of course impressive to notice that the foregoing reasoning is not restricted to gravitation only. It should hold in any kind of interaction where the wave-like clock, develops a binding, thus undertakes a mass deficiency *(without of course, loosing its "identity"),* as described above; in such a case, E_B becomes the binding energy of the wave-like clock to either field *(electric, magnetic, nuclear, gravitational, whatever)* of concern[1]. So, quite differently from the prevailing opinion, the gravitational field is not any different than other fields, in affecting the clocks. Thus, one can establish the following *simple theorem,* generalizing the previous one [1].

Theorem 3: A wave-like clock interacting with any field, electric, nuclear, gravitational, or else *(without loosing its "identity"),* retards as much as its binding energy, developed in this field.

Let us now elaborate on the binding energy.

4 Elaboration on the gravitation binding energy

In calculating the binding energy E_B, at the level of Eq.(2), we had tacitly assumed that the wave-like clock of original mass m_0, looses only an *insignificant part* of it, through the binding process. Otherwise, Eq.(7) should be written as follows:[1]

$$E_B(r) = G\mathrm{M}_0 \int_r^{\infty} \frac{m_0 c^2 - E_B(r')}{r'^2\, c_0^2} dr' , \tag{7}$$

which leads to the differential equation

$$-\frac{dE_B(r)}{dr} + \frac{G\mathrm{M}_0 E_B(r)}{r^2 c_0^2} = \frac{G\mathrm{M}_0 m_0}{r^2} , \tag{8}$$

and finally to

$$E_B(R_0) = m_0 c_0^2 \left[1 - \exp\left(-\frac{G\mathrm{M}_0}{c_0^2 R_0} \right) \right] = m_0 c_0^2 \left[1 - e^{-\alpha(R_0)} \right], \tag{9}$$

at a distance R_0 from the centre of the host celestial mass M_0, via the usual definition

$$\alpha = \alpha(r) = \frac{G\mathrm{M}_0}{c_0^2 r} \ . \tag{10}$$

The outcome E_B of Eq.(9) is zero when m_0 is at infinity; E_B becomes more and more important as α increases. Yet there appears to be *no singularity* at all *(unless* m_0 *when transplanted nearby* M_0*, is somehow degenerated).* This seems to be remarkable, since (based on Theorems 1 and 2) it yields *no singularity in time,* thus no "*black holes".*

Note that Eq.(9), along Eqs. (3) and (5) specifies how the *rest mass (measured in empty space) (or the proper mass)* is altered in the gravitational field of concern:

$$m(r) = m_0 e^{-\alpha} \; . \tag{11-a}$$

Based on Theorem 1, one can right away write how accordingly (via quantum mechanics) the proper period of time T_0 [(2)] and the proper size R_0 are altered in the gravitational field of concern:

$$T(r) = T_0 e^{\alpha} , \tag{11-b}$$

$$R(r) = R_0 e^{\alpha} . \tag{11-c}$$

Thence it is primordial to note that the quantity, mass x size^2 x period^{-1}, thus the Planck Constant, remains as an invariant, just the way the special theory of relativity requires (*whereas this universal constant does not stay as an invariant through the classical general theory of relativity*).

As pointed out, according to our approach the classical red shift due to gravitation is nothing else, but an overall mass decrease of the emitter, though there happens to be a slight discrepancy between the classical prediction (made by the general theory of relativity), and the one we established above [Eq. (11-a)].

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(2) Note that the general theory of relativity predicts [2] $T(r) = \frac{T_0}{\sqrt{1-2\alpha}}$.

This outcome to the first approximation is strikingly the same as that of Eq. (11-b).

We would like to say few words about how we come to a *mass decrease* in a gravitational field, instead of a *concluding mass increase* considered by the *general theory of relativity.*

Einstein's *depart point* is, based on the *equivalence principle,* a *mass increase* displayed by the object, carried away by the *"accelerating elevator".*

This *depart point,* though a striking idea, seems somewhat inappropriate for *(chiefly, next to the reason we will develop herein, about the validity of the principle of equivalence),* a major reason; it is that, there is a *clear asymmetry* between the *accelerating elevator* and the *gravitational field,* with respect to a distant observer.

Indeed *"Getting on the accelerating elevator" (when we are nearby at rest, in empty space)* and *"getting on a celestial body" (from empty space),* are not at all the same process, for *the distant observer*, clearly at least for one thing, i.e. he has to get *accelerated* to be able to catch up with the accelerating elevator, whereas he has to get *decelerated* in order to be able to land on the celestial body.

The first process (within the context of the theory of relativity) yields a mass increase, whereas the second one, through the line we followed, should lead to a mass decrease (with respect to the distant observer).

A mass decrease, through Theorem 2, yields a unit time increase, but also a length loosening (not a length contraction).

Thence according to the approach developed herein, *Einstein's transposition,* of *mass increase* and a concurrent *length contraction* taking place in an accelerating elevator, to a *gravitational field,* seems to be incorrect.

We shall elaborate further, on this, below.

Nonetheless Eqs. (2), (9) and (11-a), happen to be in close agreement(3) with the *gravitational potential* furnished by the general theory of relativity [23]. How come?

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(3) The gravitational potential $V(r)$, in the vicinity of a celestial body of mass M_0, furnished by the general theory of relativity is

$$V(r) = -\frac{GM_0}{r} + \frac{G^2M_0^2}{r^2c_0^2} \quad (\textit{furnished by the classical general theory of relativity}), \qquad \text{(i)}$$

whereas Eq.(2), together with Eq.(5), furnishes

$$V(r) = -\frac{E_B}{m_0} = -\frac{GM_0\chi}{r} = -\frac{GM_0}{r}\left(1-\frac{GM_0}{rc_0^2}\right) = -\frac{GM_0}{r} + \frac{G^2M_0^2}{r^2c_0^2}\,. \qquad \text{(ii)}$$

As we shall detail below, briefly for one thing it seems that, assuming the equality of the inertial mass and the gravitational mass, and overlooking the mass equivalence of the gravitational energy, constitute effects of about the same magnitude and amazingly overall cancelling each other; this should be how we could reproduce practically the same result as that of Einstein, in regards to the gravitational potential, the precession of the perihelion of Mercury, etc. Recall anyway that even alike predictions made by the general theory of relativity and the theory presented herein, are not exactly the same.

5 The general equation of gravitation motion in scalar form

Now, we are ready to derive the general equation of gravitational motion.

The idea behind it, is stunningly simple, and is rooted to the postulate, stated above. When an object enters into interaction with a celestial body, its *"total energy" (as conceived within the frame of the special theory of relativity),* throughout, must remain the same.

The extra kinetic energy it shall acquire or it shall lose on the way, thus ought to be accounted by an equivalent change in its rest mass.

Henceforth, when an object *falling* in a gravitational field, is stopped and the kinetic energy, it would have acquired is taken away, its rest mass *(as measured in empty space)* should be decreased as much as the binding energy it would have developed in the field.

Here, to make things easier, we tacitly assume that, one of the interacting objects is very massive, and the other is very small, so that we have to worry about only the small one. The one which is massive undergoes practically *no change.* The approach presented herein though, can be easily extended to the general case.

In order to ease our dissertation we shall work on a concrete basis, more specifically we will consider the planet Mercury, in motion around the sun *(without though, any loss of generality).*

(*furnished, within the given approximation, by the theory presented herein*) (*c.q.f.d.*) Note that the above expression can further be elaborated by letting the mass of the object of concern vary under the integral operation in Eq.(2). The resulting binding energy E_B, turns out to be the RHS of Eq.(9), presented above; accordingly the gravitational potential $V(r)$ becomes $V(r) = -\frac{E_B}{m_0} = -\frac{GM_0}{r} + \frac{1}{2}\frac{G^2M_0^2}{r^2c_0^2} - \frac{1}{6}\frac{G^3M_0^3}{r^3c_0^4}$ (rigorously furnished by the theory presented herein (iii)

We can conceive Mercury's motion *(around the sun),* as made of two steps:

i) Bring it from infinity to a given location, situated on its *"elliptical"* orbit around the sun; the energy this process requires, is the *magnitude of the classical potential energy.*

ii) Deliver to it, the *kinetic energy* it would display on this location. *(Note that classically, on the orbit the Newtonian total energy, i.e. the potential energy + the kinetic energy, is a constant of the motion.)*

Let us then make the following *casual* definitions.

r_0 or $r_0(t_0)$:	distance of the sun to the planet, at time t_0 *(measured in terms of the local metric)*
$m_{0\infty}$:	the planet's rest mass at infinity
$m_0(r_0)$ or $m_0(t_0)$:	the planet's rest mass at a distance r_0, or at the corresponding time t_0, as referred to the sun $m_{0\gamma}(r_0)$ or $m_{0\gamma}(t_0)$: the planet's total relativistic mass *(which is its mass at infinity decreased as much as its binding energy, but on the other hand, increased based on the special theory of relativity, due to its "translational" motion on the orbit)* at r_0, and at the corresponding time t_0
v_0 or $v_0(t_0)$ or $v_0(r_0)$:	magnitude of the tangential velocity of the planet on the orbit, at r_0, and at the corresponding time t_0, as referred to the local observer
c_0:	the velocity of light in empty space *(free of any gravitational field)*
α_0 or $\alpha_0(r_0)$:	dimensionless quantity defined (*still in terms of the local metric*) along Eq.(10), for the distance r_0 of the planet from the sun

Equation of Motion of Mercury as Assessed by the Local Observer

On any given natural orbit, the *relativistic total energy* of the object of concern, i.e $m(r_0)c_0^2$, thus $m(r_0)$ must remain constant. If the orbit is not circular, throughout the object's journey on the orbit, however this may be, both r_0 and v_0 shall vary; but $m_{0?}(r_0)c_0^2$, thus $m(r_0)$ must stay constant.

Thus starting with the energy conservation postulate, and the above definitions, one can now write the following equations based on, *first,* Eqs. (10) and (11-a), yielding the *decrease of the rest mass* of the planet brought from infinity, and *then,* the familiar *relativistic mass increase with tangential velocity on the orbit:*

$$m_0(r_0) = m_{0\infty} e^{-\alpha_0(r_0)} \ , \tag{12}$$

$$\frac{m_{0\gamma}(r_0)}{m_{0\infty}} = \frac{e^{-\alpha_0(r_0)}}{\sqrt{1-\dfrac{v_0^2}{c_0^2}}} = \mathsf{D}, \tag{13}$$

where D is a *constant* to be determined.

Note that $\alpha_0(r_0)$ remains as a constant in the case of a circular orbit; so is $v_0(r_0)$; thus, in this case, D is anyhow a constant. This special case does not advance us. Yet what is interesting, as we propose to study, is that D is whatsoever, a constant.

Let us explain this, a bit further.

According to our approach, $c_0^2\, m_{0\gamma}(r_0)$ *(the total relativistic energy of the planet)* ought to be constant all along Mercury's journey around the sun. As the planet *speeds up* nearby the sun, it is that, an infinitesimal part of its mass somehow *"sublimes"* into *kinetic energy,* yielding the *extra* kinetic energy *(the planet acquires as it speeds up);* as the planet *slows down* away from the sun, through its orbital motion, it is that, a portion of its kinetic energy somehow *"condenses"* onto its rest mass, on the orbit.

This *alternating process* through the motion, based on the *special theory of relativity,* anyway, makes that the planet's *total relativistic mass (i.e. the classical rest mass at infinity, decreased as much as the gravitational binding energy + the mass equivalent of the kinetic energy)* remains the same. This should be considered harmonious with the fact that the planet's *classical total energy* on the orbit is *constant.* We will soon elaborate on this point.

What is this constant? It would first be interesting to examine the case of *free fall,* where D *(as we shall see)* is interestingly unity.

Free fall

Consider an object originally at *rest,* practically at infinity,[4] and experiencing a *free fall* in a gravitational field. Let $m_{0\infty}$ *its rest mass, at infinity.* Its *binding energy* E_B, were it stopped at a *given altitude,* according to Eq.(9) is

$$E_B = m_{0\infty} c_0^2 (1 - e^{-\alpha_0}) , \tag{14}$$

Where α_0 represents the value of this quantity at the *altitude in consideration.*

The *rest mass* of the object at this altitude, according to Eq.(11), is $m_{0\infty} e^{-\alpha_0}$.

On the other hand, the object through its free fall, would *(up to the altitude of concern)* acquire the velocity v_0, yielding the overall mass $m_{0\infty} e^{-\alpha_0} / \sqrt{1 - v_0^2 / c_0^2}$, while some of its mass content, as just mentioned, is transformed into *kinetic energy.* The differrence of the corresponding energies is nothing, but the binding energy E_B [given by Eq.(14)]:

$$m_{0\infty}(1 - e^{-\alpha_0}) = \frac{m_{0\infty} e^{-\alpha_0}}{\sqrt{1 - \dfrac{v_0^2}{c_0^2}}} - m_{0\infty} e^{-\alpha_0} . \tag{15}$$

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[4] Here, we say *"practically at infinity",* since if the the falling object and the source of gravitation acting on this, were really at an *infinite distance* from each other, there would be no force, thus no action, thus no free fall. To remedy the situation, things can well be seen backward, i.e. one can propose to calculate the energy, necessary to bring the bound object at rest, from the given altitude, to infinity, just the way it is considered at the level of Eqs. (2) and (7), and this would anyway lead to the LHS of Eq.(14). The same philosophy may be considered at the level of Eq.(17), pointing to the situation where the free fall starts not at rest, but with an initial velocity, at *"infinity".* Thus we can well interpret this situation in the reverse direction, through which we could propose first to carry the bound object at rest, from the given altitude to infinity, and then deliver to it the kinetic energy, resulting from the velocity $v_{0\infty}$ coming into play.

This, right away yields *unity,* for the constant D , appearing in Eq.(13), i.e.

$$\frac{e^{-\alpha_0}}{\sqrt{1-\frac{v_0^2}{c_0^2}}}=1 \tag{16}$$

(in the case of an object in free fall, started at rest, practically at infinity).

Note that the *classical total energy,* i.e. *potential energy + kinetic energy,* through the *free fall* is conserved, which is quite harmonious, with Eq.(15).

If the falling object started at infinity with an *initial velocity* $v_{0\infty}$ *(and not at rest),* than Eq.(16) would become

$$\frac{e^{-\alpha_0}}{\sqrt{1-\frac{v_0^2}{c_0^2}}}=\frac{1}{\sqrt{1-\frac{v_{0\infty}^2}{c_0^2}}} \tag{17}$$

(in the case of an object in free fall, started with the initial velocity $v_{0\infty}$*, at infinity).*

Note that no matter what the direction of the initial velocity $v_{0\infty}$ at *infinity,* or the direction of v_0 at the given location, we associate with the object in hand, the above relationship is still valid.

Differential Equation of Motion as Assessed by the Local Observe

The constancy of D can further be easily checked and fixed for the case of Mercury, based on the actual data associated with the planet, at a given location of it, on the orbit.(5)

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(5) Based on Eq.(13), and the data regarding the perihelion and the aphelion i.e.

$$r_{0perihilion}=46.0x10^6\,km,\quad v_{0perihilion}=58.98\,x\,km/s,$$

$$r_{0aphelion}=69.8x10^6\,km,\quad v_{0aphelion}=38.86\,x\,km/s,$$

it can indeed be checked that, at any location on the orbit of Mercury, we precisely have c_0^2 (1-D^2) = 1.15x10^9 km^2/s^2.

For further simplicity we can recall that the orbit of the planet is nearly circular.

Thus, based on Eq.(13), we can write

$$\frac{e^{-\alpha_0}}{\sqrt{1-\frac{v_0^2}{c_0^2}}} = \mathrm{D} \cong 1-\alpha_0+\frac{1}{2}\frac{v_0^2}{c_0^2} \ ; \tag{18}$$

Here recall that v_0 is the tangential velocity of the planet, at the location r_0 on the orbit *(as referred to the local observer).*

Note further that

$$\alpha_0 = \frac{GM_0}{r_0 c_0^2} \cong \frac{GM_0}{\overline{r_0} c_0^2} = \overline{\alpha_0} \cong \frac{\overline{v_0^2}}{c_0^2} \cong 2.56x10^{-8} , \tag{19}$$

where (for a reason that we shall clarify right away) we associate with r_0, the quantity $\overline{r_0}$, i.e. the average distance of the planet to the sun (which happens to be the semi-grand axis of the elliptical orbit); $\overline{\alpha_0}$ is the average of α_0, and $\overline{v_0^2}$ the mean square velocity.

It is already striking that the second equality displayed by Eq.(18), under the assumptions in question (i.e. small v_0, small α_0), is nothing but, the Newton's equation of gravitational motion *(in its integral form),* relating the tangential velocity v_0 of the planet, to its distance to the sun.

The usual form of the equality of concern is [24, 25]

$$v_0^2 = G(\mathrm{M}_0 + m_{0P})\left(\frac{2}{r_0}-\frac{1}{a}\right) ; \tag{20}$$

here m_{0P} is the *classical mass of the planet* and a, the *semi-grand axis of the elliptical orbit of this;* $a = 57.9x10^6 km$, for the case of Mercury.

Throughout the approach presented herein, we have assumed the sun *infinitely big* as compared to Mercury, this being the reason for which the mass of the latter does not appear in our relationships. Below, we shall continue to set our relationships, that way.

At this stage it is interesting to note that Eq.(20) is nothing but the *classical energy conservation equation;* thus it states that, on the orbit *(classically),* the total energy of the planet, is conserved. Indeed the magnitude of the classical total energy is the energy one has to spend in order to remove the planet bearing a velocity v_0, on the orbit at a distance r_0 to the sun, from its actual position, to infinity. It is composed of, on the one hand the *potential energy,* of magnitude $G\mathrm{M}_0 m_{0P}/r_0$ *(which is the energy one has to spend in order to remove the planet of mass* m_{0P}, *at rest, from a distance* r_0 *to the sun, to infinity),* and on the other hand the *kinetic energy* $(1/2)m_{0P}v_0^2$.

Thus Eq.(20) states that the *magnitude of the classical total energy,* i.e. the sum of $G\mathrm{M}_0 m_{0P}/r_0$ and $(-1/2)\,m_{0P}v_0^2$, on the orbit, must be constant and equal to $G\mathrm{M}_0 m_{0P}/2a$.

Having started with Eq.(13), the *"relativistic energy conservation equation",* it should be natural, as well as fulfilling to land at the *"classical energy conservation equation",* for small velocities and weak gravitational fields.

Thus for Mercury, D *(considering the assumptions in question),* shall be given by

$$\mathrm{D} \cong 1 - \frac{\overline{\alpha_0}}{2} = 1 - \frac{G\mathrm{M}_0}{2c_0^2 a} . \tag{21}$$

Note that, because α_0 is small, D is very close to unity. Though the divergence, as small as $\sim 10^{-8}$ from unity, is still essential.

At any rate, following Eq.(13) (giving that the RHS of this equation, is constant), we expect that the *total differential* of $m_0(r_0)$, must vanish.

Thence, by *differentiating* Eq.(13), we arrive at the *rigorous equation,* regarding the revolution of the planet around the sun, or anything as such:[(6)]

[(6)] In the case we consider the *electron* revolving on an elliptic orbit around the nucleus, this equation [via Eqs. (3), (4) (5), (12) and (13), this time, written for the electron bound to the nucleus], in CGS unit system, becomes:

$$-\frac{Ze^2}{r_0^2}\sqrt{1-\frac{v_0^2}{c_0^2}} = m_{e0}\,\frac{1-\dfrac{Ze^2}{m_{e0}r_0c_0^2}}{\sqrt{1-\dfrac{v_0^2}{c_0^2}}}\,v_0\,\frac{dv_0}{dr_0}$$

$$-\frac{GM_0}{r_0^2}\left(1-\frac{v_0^2}{c_0^2}\right)=v_0\frac{dv_0}{dr_0}\ . \tag{22}$$

(*written by the author, in the local frame of reference*)

This relationship is interesting in many ways. First of all when v_0 *(as compared to the velocity of light)* is negligible, or similarly when α_0 is small, it reduces right away to the *classical Newton's equation of gravitational motion.* This can be checked immediately by differentiating Eq.(20), which is a scalar form of Newton's equation of gravitational motion.

Eq.(22) can further account for the *precession of the perihelion of a planet,* as well as the *deflection of light nearby a celestial body,* were it processed perspective than that of Einstein.

The predictions in question shall be elaborated in our next article.

Eq.(22) on the other hand, seems to be remarkable for velocities not negligible as compared to the velocity of light, since it indicates that in such a case, the effective attraction turns to be much smaller than the classically estimated one.

6 The general equation of gravitational motion in vector form

(Gravitation force = (overall mass)(acceleration)/ $(1-v_0^2/c_0^2)$

From a *rigorous mathematical point of view,* one may argue about the following.

- One does indeed land, from *Newton's equation of gravitational motion*written in *vectorial form,* to Eq.(20), [24, 25] thus also to Eq.(22), in the case the cruise velocity v_0 of the object in hand is

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here as usual, m_e is the electron mass at infinity, *e* the charge of the electron, and *Ze* the charge of the nucleus; The LHS of this equation displays how the Coulomb's Force is altered, as m_{e0} multiplied by the combersome ratio (thus the electron mass at infinity, decreased as much as the binding energy, but at the same time increased by the Lorrentz dilation factor), at the RHS represents the overall mass of the electron on the orbit. According to the approach presented herein, this is the correct equation which should have been written by Sommerfeld, who has considered the relativistic mass increase only, and not the mass decrease due to binding. (Including Dirac, noone else afterwards either, has seemingly given a condiration to the mass defect of the electron due to the binding).

> *small* as compared to the velocity of light. But can we really obtain from the *scalar* Eq.(22), a *corresponding equation in vector form,* similar to *Newton's (vectorial) equation of gravitational motion?*

The answer is, yes; it may be considered even trivial, if one recalls that the classical Newton's vectorial equation of gravitational motion can be well built on the basis of the (scalar) energy conservation equation, and that our derivation essentially is similar to this approach. Nevertheless, to be rigorous, we better elaborate on the question we just introduced.

Thus consider the *general case,* where the magnitude v_0, of the velocity vector $\underline{v}_0(t)$, changes continuously, all along the motion in question.

Through the infinitely small period of time dt_0, we have, as usual

$$d\underline{v}_0 = \underline{v}_0(t_0 + dt_0) - \underline{v}_0(t_0)\,. \tag{23}$$

Obviously $\underline{v}_0(t)$ and $d\underline{v}_0$ are not generally oriented in the same direction; $\underline{v}_0(t)$ is oriented along the direction of the motion on the orbit, whereas $d\underline{v}_0$ is directed toward the sun.

The *infinitesimal increase* dv_0 in the *"magnitude"* of $\underline{v}_0(t)$, i.e.

$$dv_0 = v_0(t_0 + dt_0) - v_0(t_0)\,, \tag{24}$$

is commonly different from $|d\underline{v}_0|$, the *"magnitude of the infinitesimal increase"* in $\underline{v}_0(t)$, though $|d\underline{v}_0|$ and dv_0 become equal, if the motion were a one dimensional motion.

Note that dv_0 vanishes in the case of a *circular orbit.* However, in this case our start point, i.e. Eq.(13), becomes a triviality; thence the differentiation of it, does not provide us with any additional information.

According to the definitions we have made along Eqs. (23) and (24), one can show that, [24, 25] the classical Newton's equation of gravitational motion, i.e.

$$\frac{G\mathrm{M}_0}{r_0^2}\frac{\underline{r}_0}{r_0} = \frac{d\underline{v}_0}{dt_0}\,, \tag{25}$$

(the classical Newton's equation of gravitational motion in vector form)

and vice-versa.

It would be useful to provide our way, a quick proof of the *vice-versa, statement in consideration.*

Thus Eq.(26) can be *classically* written as

$$-\frac{GM_0 m_0}{r_0^2} dr_0 = -F_{0G} dr_0 = m_0 v_0 dv_0 \ ; \tag{27}$$

this equation expresses that (classically) the decrease in the potential energy and the increase in the corresponding kinetic energy are equal to each other.

Here m_0 is the classical (proper, unalterable) mass of the planet, and F_{0G} the magnitude of the gravitational force between the sun and the planet, at the given location.

But evidently, with the usual notation

$$F_{0G} dr_0 = -\underline{F}_{0G} \cdot \underline{dr}_0 \,, \tag{28}$$

given that the gravitational binding energy is path independent.[7]

In Eq.(28). In this equation $\underline{F}_{0G}$ is the gravitational force *(in vector form);* $\underline{r}_0$ is the *location vector* defined along r_0 ; $|\underline{r}_0|$ and r_0 are the same quantities; one can thus write the definitions

$$dr_0 \equiv dr_0(t_0) = r_0(t_0 + dt_0) - r_0(t_0) \,, \tag{29}$$

$$d\underline{r}_0 \equiv d\underline{r}_0(t_0) = \underline{r}_0(t_0 + dt_0) - \underline{r}_0(t_0) \,. \tag{30}$$

We shall soon recall that, one can even directly prove this statement.

The negative sign at the RHS of Eq.(28) arises from the fact that, as r_0 increases, the force counteracts, making the *cosine* of the dot product negative

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[7] i.e the energy one has to furnish to the object of concern is the same, in order to bring it form an altitude r_0 to a higher altitude $r_0 + dr_0$ regardless the path he may choose; thus this energy can be expressed as the one will furnish via following the corresponding shortest radial path dr_0, straight

(or the same, as r_0 "decreases", the force acts in speeding up the motion, making the cosine of the dot product, positive).

We now rewrite Eq.(27), dividing its both sides by dt_0:

$$\underline{F}_{0G} \cdot \underline{v}_0 = m_{0P} v_0 \frac{dv_0}{dt_0} \ , \tag{31}$$

where we made use of the usual definition of $\underline{v}_0$, i.e.

$$\underline{v}_0 = \frac{d\underline{r}_0}{dt_0} \ . \tag{32}$$

Let us multiply both sides of Eq.(31) by $d\underline{v}_0$[cf. Eq.(23)], and rearrange it:

$$\left|\underline{F}_{0G}\right| \frac{\left|\underline{v}_0\right|}{v_0} \cos\theta \frac{\left|d\underline{v}_0\right|}{dv_0} = m_{0P} \frac{\left|d\underline{v}_0\right|}{dt_0} \ ; \tag{33}$$

here θ is the angle between the vector $\underline{F}_{0G}$ *(directed toward the sun),* and the vector $\underline{v}_0$ *(tangent to the orbit).* [Bear in mind that v_0 is identical to $\left|\underline{v}_0\right|$.]

One can on the other hand, easily show that[8]

$$dv_0 = -\left|d\underline{v}_0\right| \cos(\pi\text{-}\theta) = \left|d\underline{v}_0\right| \cos(\theta), \tag{34}$$

checking well and at once the case of the *circular motion,* for which $\theta = \pi/2$, and $dv_0 = 0$; one can moreover note that this relationship also checks well the sign of dv_0 for an elliptic orbit. (Note indeed that for an elliptic orbit, one has cos(θ)<0, when dv_0<0, and cos(θ)>0, when dv_0>0.)

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[8] Note that, just likewise, one can write $dr_0 = -\left|d\underline{r}_0\right| \cos\theta$ instead of Eq.(28), and accordingly the *gravitational binding energy becomes path independent.* Recall that for an elliptic orbit, one has

cosθ<0, when dr_0 >0, and cosθ >0, when dr_0 <0 .

Furthermore $d\underline{v}_0$ is directed toward the sun, along the same direction as $\underline{F}_{0G}$. This makes that Eq.(26) written in scalar form, yields well Eq.(25) written in vectorial form (c.q.f.d.).

Based on the foregoing information, it becomes clear that, departing from Eq.(22), we can obtain the *vectorial equation*

$$\frac{GM_0}{r_0^2}\left(1-\frac{v_0^2}{c_0^2}\right)\frac{\underline{r}_0}{r_0}=\frac{d\underline{v}_0}{dt_0}\ . \tag{35}$$

(the general equation of gravitational motion written by the author, in the local frame of reference)

Unless v_0 is small, this relationship displays an amazing feature; it is that the *"classical gravitational mass"* and the *"inertial mass"* are not the same. We shall elaborate on this in what follows.

Regarding the motion of a planet around the sun, the classical energy conservation approach yields well Newton's second law of motion [cf. Eq.(25)], i.e.

Gravitational Force = m_{0_p} x Acceleration,
(written out of Newtonian approach, based on energy conservation)

or the same,

Gravitational Field (Vector) = Acceleration (Vector).
(written out of Newtonian approach, based on energy conservation)

The approach presented herein, via the relativistic energy conservation, clearly, does not yield Newton's second law of motion; it yields something else.

In order to draw a *one to one* comparison between the frame we just sketched [through Eqs. (25) – (35)], and our approach, we would like to rewrite Eq.(22), out of Eq.(13), and reexamine it:

$$-\frac{GM_0}{r_0^2}\frac{m_{0\infty}e^{-\alpha_0}}{\sqrt{1-\frac{v_0^2}{c_0^2}}}dr_0=-\frac{m_{0\infty}e^{-\alpha_0}\frac{-2v_0dv_0}{2\sqrt{1-\frac{v_0^2}{c_0^2}}}}{\left(1-\frac{v_0^2}{c_0^2}\right)}. \tag{36}$$

[Eq.(22), rewritten by differentiating Eq.(13)]

The LHS of this equation expresses the infinitesimal change in the gravitational binding energy of the object in motion (with an overall mass equal to $m_{0\infty}e^{-\alpha_0}/\sqrt{1-v_0^2/c_0^2}$.)

The RHS conversely expresses the infinitesimal change in the kinetic energy of this *"overall mass", recall that* $m_{0\infty}e^{-\alpha_0}/\sqrt{1-v_0^2/c_0^2}$; overall remains constant throughout [cf. Eq.(13)]. Note further that the change on the kinetic energy, is solely due to the change on the velocity.

Thence by rereading Eq.(36), along the derivation of Newton's equation of gravitational motion [Eq.(26)], we can state that

$$\text{GravitationalForce}=\frac{\text{(OverallMass)(Acceleration)}}{\left(1-\frac{v_0^2}{c_0^2}\right)}\ ; \tag{37}$$

(the general equation of gravitational motion written by the author)

here the gravitational force, next to the sun's mass *(assumed at rest),* embodies the overall mass, $\mathrm{m}_{0\infty}\mathrm{e}^{-\alpha_0}/\sqrt{1-\mathrm{v}_0^2/\mathrm{c}_0^2}$ of the revolving object.

Eq.(37), reduces to Eq.(22), once one divides both sides by the *overall mass.*

Eq.(37), based on the analysis made on Eq.(36), seems the natural way of presenting our result. Accordingly one uses the same mass, i.e. $m_{0\infty}e^{-\alpha_0}/\sqrt{1-v_0^2/c_0^2}$, to multiply both the *gravitational field vector* and the *acceleration vector.* But then Newton's equation of gravitational motion, i.e. [Force = m_{0P} x Acceleration] is broken.

Formally, this can be saved if instead, we choose to alter the *"classical gravitational force"*; but then the *gravitational mass* and *the inertial mass,* as *classically* defined, shall not be same.

We conclude on this below.

7 Conclusion

The essence of this article was, based the energy conservation, in the broader sense of the concept, embodying the *equivalence of mass and energy,* as implied by the special theory of relativity, to derive a general equation of gravitational motion, more specifically

$$\frac{G\mathrm{M}_0}{r_0^2}\left(1-\frac{v_0^2}{c_0^2}\right)\frac{\underline{r}_0}{r_0}=\frac{d\underline{v}_0}{dt_0} \ . \qquad (35)$$

(the general equation of gravitational motion written by the author, in the local frame of reference)

This becomes the Newton's equation of motion, only if v_0 is small as compared to the velocity of light. In our next article, we shall see, how this equation can cover up the basic predictions envisaged by the general theory of relativity, provided that one takes into consideration the fact that the *mass deficiency* due to the *binding,* alters via *quantum mechanics, unit lengths, unit periods of time,* etc, along Theorem 1, presented above.

The way it stands though, *the principle of equivalence about the gravitational mass and the inertial mass,* in general, seems inadequate.

This principle is anyway severely questioned [26-28].

Nonetheless we can *formally* save Newton's equation of gravitational motion, by redefining the *gravitational mass.*

Thus consider the classical formulation of Newton's equation of gravitational motion, tuned along the special theory of relativity, *i.e.* with the familiar notation [15].

Classically expressed Gravitational Force
= [**d** *(Momentum of the object in motion, due to gravitation)* /$\mathbf{dt}_0$]. (38)

To ease our expression, let us continue to consider, say *Mercury* of mass $m_{0\infty}$, defined at *infinity,* in its motion around the sun of mass M_0, without however any *loss of generality.*

Note that here it is assumed that we are positioned locally. Things will be seen differently, when we will be positioned at a distance far away from the sun's gravitational field. This latter situation shall be undertaken in our next article.

Comparing Eqs. (35) and (36), the mass m_{0I}, pertaining to the planet, and entering the formulation of *the momentum of the planet,* shall be $m_{0\infty}e^{-\alpha_0}/\sqrt{1-v_0^2/c_0^2}$; this corresponds to the *classical inertial mass;* it is a *constant* of our approach, therefore it comes out of the *differentiation operation* on the momentum.

Let us then call m_{0G}, a *gravitational mass* pertaining to the planet, taking part in the usual *gravitational force* acting between the *sun* and the *planet,* so that

$$G\frac{\mathrm{M}_0 m_{0G}}{r_0^2}\frac{\underline{r}_0}{r_0}=m_{0I}\frac{d\underline{v}_0}{dt_0}=\frac{m_{0\infty}e^{-\alpha_0}}{\sqrt{1-\frac{v_0^2}{c_0^2}}}\frac{d\underline{v}_0}{dt_0}. \tag{39}$$

[Eq.(38) written via the introduction of a gravitational mass]

This latter equation becomes the same as Eq.(35), if we propose to write

$$m_{0G}=m_{0\infty}e^{-\alpha_0}\sqrt{1-\frac{v_0^2}{c_0^2}} \quad . \tag{40}$$

(gravitational mass that would take part in the classical gravitational force expression, as assessed by the local observer)

Our result, at any rate, leads us to a straightforward conclusion, albeit totally against the prevailing wisdom; it is worth to state it as a separate theorem.

Theorem 4: The gravitational mass m_{0G}, and the inertial mass m_{0I}, as *classically* defined, are not the same; the theory presented herein, to *formally* save Newton's equation of gravitational motion, predicts

$$m_{0G}=m_{0\infty}e^{-\alpha_0}\sqrt{1-\frac{v_0^2}{c_0^2}},$$

given that

$$m_{0I} = \frac{m_{0\infty} e^{-\alpha_0}}{\sqrt{1 - \frac{v_0^2}{c_0^2}}} \;;$$

though undetectable, for most cases we observe, m_{0G} and m_{0I} differ.

The equality of the gravitational mass and the inertial mass, based on the approach presented herein is an approximation which is acceptable, only if the velocity of the object in motion is small, as compared to the velocity of light in empty space.

It is interesting to note that, all the *highly precise measurements* regarding the relative divergence of these two masses, are performed on Earth *(where the observer is moving with Earth),* so that the *precision* they produce, no matter how fine this may be, should be considered, as misleading. In effect, since the gravitational mass, as stated by Eq.(40), depends on the velocity, one should not rely on the experiments in question, any more then he should count on the null result of the Michelson Morley experiment [29] *(which, being performed on Earth, fails to detect the motion of Earth around the sun, or else).* In other terms, the *principle of relativity (the main ingredient of the special theory of relativity),* forbids that we can on Earth, detect any such difference, based on the *velocity of motion* in question *(since otherwise we should be able to tell accordingly, how fast we are cruising, or rotating in space, and we cannot).*

Not knowing that the equality of gravitational mass and inertial mass, is only approximate, one may still insist *(just the way it is done regarding the experiments in question)* that, such an equality can well be established on Earth. But the rotational velocity v_0 of Earth around itself is 1667 km/hour. Hence one should attain a precision of v_0^2/c_0^2, i.e. better than 2.6 x 10^{-12}, whereas the highest precision reached so far, is bearly, this much.

On the other hand, measurements based on a possible polarization of Earth and the Moon, through their motion around the sun *(on which, as Newton himself predicted, we can indeed rely),* require a precision of $\sim 10^{-8}$ (as the related ratio of v_0^2 to c_0^2), whereas the precisions actually reached ($\sim 10^{-4}$), happens to be far below this [30, 31].

Note further that, even through the fastest observable celestial motions, such as that of binary stars, around each other *(where the objects move with speeds around 1000 km/s),* the difference between the gravitational mass and the inertial mass, remains still undetectable.

In contrast, it is astoundingly interesting to note that Eq.(22) can be obtained from the following equation bearing the same form as that of *the classical Newton Equation of Motion,* i.e. Eq.(26):

$$-\frac{G\mathrm{M}_0(m_{0\infty}e^{\alpha_0})}{r_0^2}=v_0\frac{d(m_{0\infty}e^{\alpha_0}v_0)}{dr_0}\ ,\tag{41}$$

This can be interpreted in the following way; if the "*local mass*" m_{oL} were even given by

$$m_{0L}=m_{0\infty}e^{\alpha_0}\ ,\tag{42}$$

instead of that given by Eq.(11-a) (i.e. $m_{0L}=m_{0\infty}e^{-\alpha}$), and if the local relativistic effect due to the translational motion of the object of concern could be ignored [since, at this stage, the momentum quantity, expressed as $m_{0\infty}\,e^{\alpha_0}v_0$, under the differentiation operation at the RHS of Eq.(41) clearly does not cover the effect due to the translational motion of the object], *only then* we could claim that the principle of equivalence holds.

In other words, only in such a case, and in conformity with Eq.(38), the gravitational mass (or mass taking place next to the other one, within the frame of the gravitational force expression, i.e. here) and the inertial mass (or mass taking place in the momentum expression entering the derivation operation with respect time, i.e. here, one again $m_{0\infty}\,e^{\alpha_0}v_0$), can be considered the same[(9)].

But this is not the case; that is, through the approach presented herein, Eq.(42) is *incorrect;* furthermore the *local relativistic effect due to the translational motion of the object,* should be considered as essential on the basis of a relativistic approach, thus cannot be overlooked.

Henceforth (according to the approach presented herein), the *principle of equivalence* must be incorrect.

Recall further that Eq.(35) on the other hand, seems to be remarkable for velocities not negligible as compared to the velocity of light, since it indi-

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(9) We had already figured out that they are the same, if v_0 is relatively small. However here it is question of a different category, since Eq.(41), yields well Eq.(22), whether v_0 is small or not.

cates that in such a case, the effective attraction turns to be much smaller than the classically estimated one.

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